Theorem. *For each integer $n > 1$, there exist a constant integer d, such that*

$$a^{k+\phi(n)} \equiv a^k \pmod{n}$$

for any $a \in \mathbb{Z}$ and $k \geq d$.

Proof. *Let $n = \prod_{i=1}^{r} p_i^{e_i}$ be the factorization of n into primes, take $d = \max\{e_i\}$. For each p_i, we need to show that $p_i^{e_i} | a^k(a^{\phi(n)} - 1)$. Let $q_i = p_i^{e_i}$, when $(a, p) = 1$, we have $(a, q_i) = 1$, $a^{\phi(q_i)} = 1 \pmod{q_i}$, therefore $a^{\phi(n)} = 1 \pmod{q_i}$. Otherwise, $a = a_1 p$, then $p_i^{e_i} | a^k$ since $k \geq e_i$.*